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Hadron Magnetic Moment in the Spinor Strong Interaction Model

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Hadron magnetic moments are considered in the framework of the spinor strong interaction theory for hadron spectra proposed by the author. Expressions of magnetic moments of ground-state hadrons are derived. These differ from the conventional ones in that they are no longer phenomenological and are basically relativistic. Pseudoscalar mesons have no magnetic moment. Charged vector meson magnetic moment values are given. The magnetic moment operators operate in the internal space, so that the ground-state octet baryons have the same spin-space symmetry, including the Λ. A formula for the ground-state octet baryon magnetic moment is derived from the basic spinor strong interaction baryon equations previously given, essentially without approximation and in a way analogous to the way in which the electron magnetic moment is derived.

1. INTRODUCTION

One of the main successes of the quark model of hadrons of the 1960s is its semiquantitative account of the ground-state octet baryon magnetic moments (Lichtenberg, 1978, Particle Data Group, 1992). The approach is, however, basically phenomenological and nonrelativistic. Quark magnetic moments are assumed to be eigenvalues of Dirac magnetic moment operators in spin space. Baryon magnetic moments are obtained as expectation values of the sum of three such operators, one for each quark, over the baryon spin states. Although numerical agreement is acceptable at this level, there are two basic difficulties.

In the first place, the spin symmetry of the Λ has to be different from that of the other seven octet baryon members in order to distinguish it

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from the Σ^0. This also holds for some charmed baryons. Second, the effect of quark motion on baryon magnetic moments is excluded. Quantum Chromodynamics (QCD) introduced in the mid-1970s as a theory for strong interactions has been unsuccessful in making progress and improving upon predictions in this connection.

The purpose of this paper is to remove these two basic difficulties and present derivations of hadron magnetic moments essentially without approximation in a way analogous to that via which the electron magnetic moment is obtained. This is carried out in the framework of the recently proposed spinor strong interaction formalism for mesons (Hoh, 1993) and baryons (Hoh, 1994*a*) (hereafter referred to as I and II, respectively). The so-obtained magnetic moment formulas are no longer phenomenological and are basically relativistic.

This spinor strong interaction formalism differs fundamentally from the conventional QCD-oriented approaches. It is so named because it is based upon manipulations of van der Waerden's spinor having two components rather than Dirac's bispinor, which contains four components. Linear confinement for mesons and harmonic confinement for baryons arise naturally from the covariant basic equations without approximation. Radial equations for hadron wave functions in the relative space of the quarks are given. Gauge invariance of the meson equations further predicts that the Higgs particle is unnecessary, resolves the so-called $U(1)$ problem, and links strong and electronweak interactions (Hoh, 1994*b*).

In Section 2, the basic meson equations of I are extended to include an external electromagnetic field. The basic equation for obtaining meson magnetic moments is derived. Considering the magnetic moment energy to be a perturbation, the zeroth-order solutions to the meson equations given in I are briefly reviewed in Section 3. Meson magnetic moments are then obtained in Section 4.

In Section 5, the basic baryon equations of II are modified and analogously extended to include an electronmagnetic field. A basic equation for obtaining baryon magnetic moments is derived. This equation is further treated and prepared in Section 6 so that a formula for the ground-state octet baryon magnetic moments is obtained in Section 7. In Section 8, this formula is expressed in terms of radial baryon wave functions which contains the effects of quark motion and masses. Dropping such motion, the "static quark" octet baryon magnetic moments are evaluated.

2. BASIC MESON EQUATIONS

To consider the meson magnetic moment, the basic quark equations (I5.1)–(I5.2) are generalized to include a $U(1)$ gauge field via

$$\partial_{\mathrm{I}}^{a\dot{b}} \to D_{\mathrm{I}}^{a\dot{b}} = \partial_{\mathrm{I}}^{a\dot{b}} + iq_{\mathrm{op}}(z_{\mathrm{I}}, \partial/\partial z_{\mathrm{I}})A^{a\dot{b}}(x_{\mathrm{I}}) \tag{2.1a}$$

$$\partial_{\mathrm{II}\dot{e}f} \to D_{\mathrm{II}\dot{e}f}^{*} = \partial_{\mathrm{II}\dot{e}f} - iq_{\mathrm{op}}^{*}(z_{\mathrm{II}}, \partial/\partial z_{\mathrm{II}})A_{\dot{e}f}(x_{\mathrm{II}}) \tag{2.1b}$$

where A denotes an external electromagnetic field. The constant charges multiplying it have been generalized to internal operators operating upon the quark internal functions dependent upon the complex internal coordinates z_{I} and z_{II} associated with the quark and antiquark at the space-time coordinates x_{I} and x_{II}, respectively. The symbols are defined in I.

Equations (I5.1)–(I5.2) generalized according to (2.1) are multiplied together following the procedure of Section 5 of I. The resulting generalized meson equations that replace (I5.4) read

$$D_{\mathrm{I}}^{a\dot{b}}\chi_{\dot{b}}^{f}(x_{\mathrm{I}}, x_{\mathrm{II}})\xi^{p}{}_{r}(z_{\mathrm{I}}, z_{\mathrm{II}})D_{\mathrm{II}f\dot{e}}^{*} = (\phi_{P}(x_{\mathrm{I}}, x_{\mathrm{II}}) - m_{2\mathrm{op}})\psi_{\dot{e}}^{a}(x_{\mathrm{I}}, x_{\mathrm{II}})\xi^{p}{}_{r}(z_{\mathrm{I}}, z_{\mathrm{II}})$$
$$\tag{2.2}$$
$$D_{\mathrm{I}\dot{c}b}\psi_{\dot{e}}^{b}(x_{\mathrm{I}}, x_{\mathrm{II}})\xi^{p}{}_{r}(z_{\mathrm{I}}, z_{\mathrm{II}})D_{\mathrm{II}}^{*\dot{e}d} = (\phi_{P}(x_{\mathrm{I}}, x_{\mathrm{II}}) - m_{2\mathrm{op}})\chi_{\dot{c}}^{d}(x_{\mathrm{I}}, x_{\mathrm{II}})\xi^{p}{}_{r}(z_{\mathrm{I}}, z_{\mathrm{II}})$$

Here, χ and ψ are the space-time meson wave functions, $\xi^{p}{}_{r}$ the normalized internal meson functions, p and r the quark and antiquark flavors, respectively, $m_{2\mathrm{op}}$ the internal mass-squared operator (I5.3b), and ϕ_P the quark–antiquark pseudoscalar interaction (I4.8b). Equation (2.2) is invariant under the $U(1)$ gauge transformation

$$\begin{aligned} \chi_{\dot{b}}^{e}(x_{\mathrm{I}}, x_{\mathrm{II}}) &\to \chi_{\dot{b}}^{e}(x_{\mathrm{I}}, x_{\mathrm{II}}) \exp(iq_{\mathrm{op}}(z_{\mathrm{I}}, \partial/\partial z_{\mathrm{I}})\varphi_{\mathrm{q}}(x_{\mathrm{I}})) \\ A^{a\dot{b}}(x_{\mathrm{I}}) &\to A^{a\dot{b}}(x_{\mathrm{I}}) - \varphi_{\mathrm{I}}^{a\dot{b}}\partial_{\mathrm{q}}(x_{\mathrm{I}}) \end{aligned} \tag{2.3}$$

where φ_{q} is a local phase. Multiplication of (2.2) by $\xi_{p}{}^{r}(z_{\mathrm{I}}, z_{\mathrm{II}})$, the complex conjugate of the internal meson functions $\xi^{p}{}_{r}(z_{\mathrm{I}}, z_{\mathrm{II}})$, leads to

$$\begin{aligned} &(\partial_{\mathrm{I}}^{a\dot{b}} + iq_{\mathrm{I}}A^{a\dot{b}}(x_{\mathrm{I}}))\chi_{\dot{b}}^{f}(x_{\mathrm{I}}, x_{\mathrm{II}})(\partial_{\mathrm{II}f\dot{e}} + iq_{\mathrm{II}}A_{f\dot{e}}(x_{\mathrm{II}})) \\ &\quad = (\phi_{P}(x_{\mathrm{I}}, x_{\mathrm{II}}) - M_{m}^{2})\psi_{\dot{e}}^{a}(x_{\mathrm{I}}, x_{\mathrm{II}}) \end{aligned} \tag{2.4a}$$

$$\begin{aligned} &(\partial_{\mathrm{I}\dot{c}b} + iq_{\mathrm{I}}A_{\dot{c}b}(x_{\mathrm{I}}))\psi_{\dot{e}}^{b}(x_{\mathrm{I}}, x_{\mathrm{II}})(\partial_{\mathrm{II}}^{\dot{e}d} + iq_{\mathrm{II}}A^{\dot{e}d}(x_{\mathrm{II}})) \\ &\quad = (\phi_{P}(x_{\mathrm{I}}, x_{\mathrm{II}}) - M_{m}^{2})\chi_{\dot{c}}^{d}(x_{\mathrm{I}}, x_{\mathrm{II}}) \end{aligned} \tag{2.4b}$$

Here,

$$q_{\mathrm{I}} = \xi_{p}{}^{r}(z_{\mathrm{I}}, z_{\mathrm{II}})q_{\mathrm{op}}(z_{\mathrm{I}}, \partial/\partial z_{\mathrm{I}})\xi^{p}{}_{r}(z_{\mathrm{I}}, z_{\mathrm{II}}) \tag{2.5a}$$

$$q_{\mathrm{II}} = -\xi_{p}{}^{r}(z_{\mathrm{I}}, z_{\mathrm{II}})q_{\mathrm{op}}^{*}(z_{\mathrm{II}}, \partial/\partial z_{\mathrm{II}})\xi^{p}{}_{r}(z_{\mathrm{I}}, z_{\mathrm{II}}) = q_{\mathrm{II}}^{*} \tag{2.5b}$$

$$\xi_{p}{}^{r}(z_{\mathrm{I}}, z_{\mathrm{II}})\xi^{p}{}_{r}(z_{\mathrm{I}}, z_{\mathrm{II}}) = 1 \tag{2.6}$$

z_{I} and z_{II} denote z_{I}^{p} and $z_{\mathrm{II}r}$, etc., and are normalized complex vectors satisfying the orthonormality relations

$$z_{\mathrm{I}}^{p} z_{\mathrm{I}r} = z_{\mathrm{II}}^{p} z_{\mathrm{II}r} = \delta^{p}{}_{r}, \qquad z_{\mathrm{I}}^{p} z_{\mathrm{II}r} = 0 \tag{2.7}$$

The quark charge operator, analogous to the quark mass operator of (I9.3a), is of the form

$$q_{\mathrm{op}}(z_{\mathrm{I}}, \partial/\partial z_{\mathrm{I}}) = \sum_{v} q_{v}(z_{\mathrm{I}}^{v}\, \partial/\partial z_{\mathrm{I}}^{v} - z_{\mathrm{I}v}\, \partial/\partial z_{\mathrm{I}v}) \tag{2.8a}$$

where q_v is the charge of a quark with flavor v;

$$q_1 = q_4 = 2e/3, \qquad q_2 = q_3 = q_5 = -e/3 \tag{2.8b}$$

Equation (2.8a) also holds for $\mathrm{I} \to \mathrm{II}$.

Operation of (2.4a) by the operator on the left of (2.4b) yields

$$\begin{aligned}(\partial_{\mathrm{I}\dot{c}a} &+ iq_{\mathrm{I}} A_{\dot{c}a}(x_{\mathrm{I}}))(\partial_{\mathrm{I}}^{a\dot{b}} + iq_{\mathrm{I}} A^{a\dot{b}}(x_{\mathrm{I}})) \\ &\times (\partial_{\mathrm{II}}^{d\dot{e}} + iq_{\mathrm{II}} A^{d\dot{e}}(x_{\mathrm{II}}))(\partial_{\mathrm{II}\dot{e}f} + iq_{\mathrm{II}} A_{\dot{e}f}(x_{\mathrm{II}}))\chi^{f}_{\dot{b}}(x_{\mathrm{I}}, x_{\mathrm{II}}) \\ &= (\phi_{P}(x_{\mathrm{I}}, x_{\mathrm{II}}) - M_{m}^{2})\chi^{d}_{\dot{c}}(x_{\mathrm{I}}, x_{\mathrm{II}}) + R'_{M}\end{aligned} \tag{2.9a}$$

$$R'_{M} = [\phi_{P}(x_{\mathrm{I}}, x_{\mathrm{II}}), (\partial_{\mathrm{I}\dot{c}a} + iq_{\mathrm{I}} A_{\dot{c}a}(x_{\mathrm{I}}))(\partial_{\mathrm{II}}^{d\dot{e}} + iq_{\mathrm{II}} A^{d\dot{e}}(x_{\mathrm{II}}))\psi^{a}_{\dot{e}}(x_{\mathrm{I}}, x_{\mathrm{II}})] \tag{2.9b}$$

where $[A, B] = AB - BA$ and M_m^2 is the eigenvalue of $m_{2\mathrm{op}}$ of (I5.5).

The external field A produces an effect on the quarks that is in general much weaker than that due to the interquark potential ϕ_P. It can therefore be regarded as a first-order perturbation.

3. ZEROTH-ORDER SOLUTIONS TO THE MESON EQUATIONS

Neglecting at first the electromagnetic field A in (2.4), we treated the zeroth-order meson equations in the rest frame in I. Some relevant notations and results are reproduced below for reference. First, χ is decomposed into a singlet and a triplet, and laboratory and relative coordinates are introduced. We have

$$\chi_{\dot{b}f}(x_{\mathrm{I}}, x_{\mathrm{II}}) = \chi'_{0}(x_{\mathrm{I}}, x_{\mathrm{II}})\delta_{\dot{b}f} + \boldsymbol{\sigma}_{\dot{b}f}\boldsymbol{\chi}'(x_{\mathrm{I}}, x_{\mathrm{II}}) \tag{3.1}$$

$$X = X^{\mu} = (1 - a_m)x_{\mathrm{I}}^{\mu} + a_m x_{\mathrm{II}}^{\mu}, \qquad x = x^{\mu} = x_{\mathrm{II}}^{\mu} - x_{\mathrm{I}}^{\mu} \tag{3.2}$$

where a_m is a constant. A plane wave ansatz is made,

$$\chi'_{0}(x_{\mathrm{I}}, x_{\mathrm{II}}) = e^{-iK_{\mu}X^{\mu}}\chi'_{0}(x), \qquad \boldsymbol{\chi}'(x_{\mathrm{I}}, x_{\mathrm{II}}) = e^{-iK_{\mu}X^{\mu}}\boldsymbol{\chi}'(x) \tag{3.3}$$

where $K_\mu = (E_0, -\mathbf{K})$, E_0 is the total energy, and $\mathbf{K}$ is the momentum of the meson. The same expressions hold for $\chi \to \psi$. Solutions in the relative time

x^0 and space $\mathbf{x}$ of the form

$$\varphi(x) = e^{i\omega_0 x^0}\varphi(\mathbf{x}), \qquad \varphi = \psi'_0, \boldsymbol{\psi}', \chi'_0, \boldsymbol{\chi}' \tag{3.4}$$

are sought. Here, ω_0 is the relative energy of the quarks. The choice $a_m = 1/2 + \omega_0/E_0$ was then made. Further treatment was limited to the rest frame $\mathbf{K} = 0$ in which the a_m and ω_0 terms cancel. It was shown that $\chi'_0(\mathbf{x}) = -\psi'_0(\mathbf{x})$ represents the wave function of the pseudoscalar mesons and $\boldsymbol{\chi}'(\mathbf{x}) = \boldsymbol{\psi}'(\mathbf{x})$ that of the vector mesons.

4. MESON MAGNETIC MOMENT

Return to (2.9) and consider the meson at rest with an energy

$$E = E_0 + E_{1M} \tag{4.1}$$

where E_{1M} is due to the electromagnetic field A and hence is also of first order. Since A varies little inside the hadron, let $A^\mu(x_{\text{I}}) = A^\mu(x_{\text{II}}) = A^\mu(X)$. For simplicity, the electrostatic potential is put to zero and the vector potential is taken to be independent of the laboratory frame time, i.e., $A^\mu(X) = (0, \mathbf{A}(\mathbf{X}))$.

The pseudoscalar meson has only one wave function component $\chi'_0(\mathbf{x})$ at rest and therefore has no magnetic moment. There are no data to verify this result.

For the vector mesons at rest, the relative energy is $\omega_0 = 0$ in a quantized treatment (Hoh, 1994*b*), so that (2.9a) can be put in the form

$$\begin{aligned}
&\left(\frac{i}{2}\delta_{\dot{c}a}(E_0 + E_{1M}) - \partial_{\dot{c}a} + \frac{1}{2}\partial_{\mathbf{X}\dot{c}a} + iq_{\text{I}}A_{\dot{c}a}(\mathbf{X})\right)\\
&\quad\times\left(\frac{i}{2}\partial^{a\dot{b}}(E_0 + E_{1M}) - \partial^{a\dot{b}} + \frac{1}{2}\partial_{\mathbf{X}}^{a\dot{b}} + iq_{\text{I}}A^{\alpha\dot{b}}(\mathbf{X})\right)\\
&\quad\times\left(\frac{i}{2}\delta_{d\dot{e}}(E_0 + E_{1M}) + \partial_{d\dot{e}} + \frac{1}{2}\partial_{\mathbf{X}d\dot{e}} + iq_{\text{II}}A_{d\dot{e}}(\mathbf{X})\right)\\
&\quad\times\left(\frac{i}{2}\delta^{\dot{e}f}(E_0 + E_{1M}) + \partial^{\dot{e}f} + \frac{1}{2}\partial_{\mathbf{X}}^{\dot{e}f} + iq_{\text{II}}A^{\dot{e}f}(\mathbf{X})\right)\boldsymbol{\sigma}_{\dot{b}f}\boldsymbol{\psi}'(\mathbf{x})\\
&= (M_m^2 - \phi_{P1}(|\mathbf{x}|))^2\boldsymbol{\sigma}_{\dot{c}d}\boldsymbol{\psi}'(\mathbf{x}) + R_M
\end{aligned} \tag{4.2}$$

Here, R_M is R'_M with its indices appropriately lowered and raised, δ denotes the Kronecker delta, and ∂ and $\partial_{\mathbf{X}}$ are differentiations with respect to $\mathbf{x}$ and $\mathbf{X}$, respectively. The exponential factor of (3.3) has been removed from (4.2). Terms to first order in E_{1M} are collected to yield

$$\frac{1}{4}E_{1M}(E_0^3 + 4E_0\Delta)\boldsymbol{\sigma}_{\dot{c}d}\boldsymbol{\psi}'(\mathbf{x}) \tag{4.3a}$$

where $\Delta = (\partial/\partial\mathbf{x})^2$. Only terms to first order in A in (4.2) that split the different $\boldsymbol{\psi}'(\mathbf{x})$ components contribute to the magnetic moment. The sum of such terms is

$$\frac{1}{2}\left(\frac{1}{4}E_0^2+\Delta\right)[-q_{\rm I}(\boldsymbol{\sigma}_{\dot{c}\dot{b}}\mathbf{H})(\boldsymbol{\sigma}_{\dot{b}a}\boldsymbol{\psi}')-q_{\rm II}(\boldsymbol{\sigma}_{df}\mathbf{H})(\boldsymbol{\sigma}_{f\dot{c}}\boldsymbol{\psi}')] \qquad (4.3b)$$

where $\mathbf{H}=\mathbf{H}(\mathbf{X})=\partial_{\mathbf{X}}\times\mathbf{A}(\mathbf{X})$ is the external magnetic field. Let $\mathbf{H}(\mathbf{X})=(0,0,H_3(\mathbf{X}))$,

$$\boldsymbol{\sigma}_{\dot{c}d}\boldsymbol{\psi}'(\mathbf{x})=\begin{pmatrix}\psi_3 & \psi_-\\ \psi_+ & -\psi_3\end{pmatrix} \qquad (4.4)$$

and denote the spin component splitting parts of E_{1M} by $E_{1M\pm}$ and E_{1M3} associated with $\psi_\pm$ *and* ψ_3, respectively. Equation (4.3) yields

$$E_{1M3}\psi_3=0 \qquad (4.5a)$$

$$E_{1M\mp}(1+4\Delta/E_0^2)\psi_\mp=\pm H_3(\mathbf{X})(q_{\rm I}+q_{\rm II})(1+4\Delta/E_0^2)\psi_\mp \qquad (4.5b)$$

The Δ terms associated with quark motion cancel out and the magnetic moment of the vector meson is

$$\mu_M=\frac{q_{\rm I}+q_{\rm II}}{2E_0} \qquad (4.6)$$

The internal functions of the positively charged vector mesons are given by (I9.1a), which with (2.6), reads

$$\xi^p{}_r=\frac{1}{\sqrt{2}}(z_{\rm I}^p z_{{\rm II}r}+z_{{\rm I}r}z_{\rm II}^p) \qquad (4.7)$$

For ρ^+, K^{*+}, D^{*+}, D_s^{*+}, and B^{*+}, the corresponding flavor indices are $(p,r)=(1,2),(1,3),(2,4),(3,4)$, and $(1,5)$, respectively. With these values, (4.7), (2.5), (2.7), and (2.8) yield simply $q_{\rm I}=q_{\rm II}=1$. The magnetic moments of these vector mesons are inversely proportional to their masses and are, in units of proton magneton, 2.44, 2.1, 0.936, 0.889, and 0.352, respectively.

The negatively charged companions of these five vector mesons have magnetic moments of opposite sign. For neutral vector mesons, the upper and lower indices of each term in their internal functions are the same, as is shown for two cases in (I9.7). Therefore, $q_{\rm I}=q_{\rm II}=0$ and their magnetic moment vanishes. These results can presently not be verified, since no data exists.

5. BASIC BARYON EQUATIONS

Equations (II3.6) and (II3.8) have been proposed to account for baryon spectra. Under mutual scalar quark–quark interaction, a suitable description of baryon spectra was obtained when two of the quarks in the baryon are merged into a diquark. In this way, the basically three-body problem was reduced to a two-body one.

The quarks, however, do not have to respond to the influence of an external electromagnetic field in the same way. Instead, the three pairs of quark equations (II2.1)–(II2.3), generalized to include internal quark functions according to Section 3 of II, are further modified to include electromagnetic fields via substitutions of the type (2.1a). Applying the same multiplication and generalization procedures as in Section 2 of II without collapsing two quarks into a diquark, we obtain the generalized baryons equations

$$D_{\mathrm{I}}^{a\dot{b}} D_{\mathrm{III}}^{g\dot{h}} D_{\mathrm{II}f\dot{e}} \chi_{\dot{b}\dot{h}}{}^{f}(x_{\mathrm{I}}, x_{\mathrm{III}}, x_{\mathrm{II}}) \xi^{psq}(z_{\mathrm{I}}, z_{\mathrm{III}}, z_{\mathrm{II}})$$
$$= -i(m_{3\mathrm{op}} + \phi_S(x_{\mathrm{I}}, x_{\mathrm{III}}, x_{\mathrm{II}})) \psi^{aq}{}_{\dot{e}}(x_{\mathrm{I}}, x_{\mathrm{III}}, x_{\mathrm{II}}) \xi^{psq}(z_{\mathrm{I}}, z_{\mathrm{III}}, z_{\mathrm{II}}) \quad (5.1\mathrm{a})$$

$$D_{\mathrm{I}\dot{b}c} D_{\mathrm{III}\dot{h}k} D_{\mathrm{II}}^{\dot{e}d} \psi^{ck}{}_{\dot{e}}(x_{\mathrm{I}}, x_{\mathrm{III}}, x_{\mathrm{II}}) \xi^{psq}(z_{\mathrm{I}}, z_{\mathrm{III}}, z_{\mathrm{II}})$$
$$= -i(m_{3\mathrm{op}} + \phi_S(x_{\mathrm{I}}, x_{\mathrm{III}}, x_{\mathrm{II}})) \chi_{\dot{b}\dot{h}}{}^{d}(x_{\mathrm{I}}, x_{\mathrm{III}}, x_{\mathrm{II}}) \xi^{psq}(z_{\mathrm{I}}, z_{\mathrm{III}}, z_{\mathrm{II}}) \quad (5.1\mathrm{b})$$

which replaces (II3.6) and is the baryon counterpart of (2.2). Here, χ and ψ are the baryon wave functions, and ϕ_S is the quark–quark scalar interaction of (II2.8) without the $x_{\mathrm{III}} \to x_{\mathrm{I}}$ merging to form diquarks, analogous to the expression intervening between (II2.4c) and (II2.5a). $m_{3\mathrm{op}}$ is the internal mass-cubed operator (II3.5) generalized to contain a third internal coordinate z_{III}.

Analogous to the transition of (2.2) to (2.9), (5.1) is multiplied by $\xi_{psq}(z_{\mathrm{I}}, z_{\mathrm{III}}, z_{\mathrm{II}})$, the complex conjugate of the ξ in (5.1). Equation (2.5a) is modified for the baryon case according to

$$q_{\mathrm{I}} = \xi_{psq}(z_{\mathrm{I}}, z_{\mathrm{III}}, z_{\mathrm{II}}) q_{\mathrm{op}}(z_{\mathrm{I}}, \partial/\partial z_{\mathrm{I}}) \xi^{psq}(z_{\mathrm{I}}, z_{\mathrm{III}}, z_{\mathrm{II}}) \quad (5.2)$$

Similar equations hold for $\mathrm{I} \to \mathrm{III}$ and II in q_{op}. Analogously, (2.6) becomes

$$\xi_{psq}(z_{\mathrm{I}}, z_{\mathrm{III}}, z_{\mathrm{II}}) \xi^{psq}(z_{\mathrm{I}}, z_{\mathrm{III}}, z_{\mathrm{II}}) = 1 \quad (5.3)$$

The so-modified (5.1a) is operated upon by the operator on the left of the so-modified (5.1b) to produce

$$D'_{\mathrm{I}\dot{c}a} D_{\mathrm{I}}^{\prime a\dot{b}} D'_{\mathrm{III}\dot{d}g} D_{\mathrm{III}}^{\prime g\dot{h}} D_{\mathrm{II}}^{\prime k\dot{e}} D'_{\mathrm{II}\dot{e}f} \chi_{\dot{b}\dot{h}}{}^{f}(x_{\mathrm{I}}, x_{\mathrm{III}}, x_{\mathrm{II}})$$
$$= -(M_b^3 + \phi_S(x_{\mathrm{I}}, x_{\mathrm{III}}, x_{\mathrm{II}}))^2 \chi_{\dot{c}\dot{d}}{}^{k}(x_{\mathrm{I}}, x_{\mathrm{III}}, x_{\mathrm{II}}) + R_B \quad (5.4\mathrm{a})$$

$$R_B = -i[\phi_S(x_{\mathrm{I}}, x_{\mathrm{III}}, x_{\mathrm{II}}), D'_{\mathrm{I}\dot{c}a} D'_{\mathrm{III}\dot{d}g} D_{\mathrm{II}}^{\prime k\dot{e}} \psi^{ag}{}_{\dot{e}}(x_{\mathrm{I}}, x_{\mathrm{III}}, x_{\mathrm{II}})] \quad (5.4\mathrm{b})$$

where

$$D'_{\mathrm{I}\dot{c}a} = \partial_{\mathrm{I}\dot{c}a} + iq_{\mathrm{I}} A_{\dot{c}a}(x_{\mathrm{I}}) \quad (5.5)$$

together with similar relations for $\mathrm{I} \to \mathrm{III}$ and II. Further, M_b^3 is the eigenvalue of m_{3op} and can be verified to have the same values as in (II8.7), (II8.11b) and (II8.11c) obtained with the $z_{III} \to z_I$ merging there.

6. FURTHER DEVELOPMENT OF THE BARYON EQUATIONS

Introduce the baryon laboratory coordinate X and quark–quark relative coordinates x and y:

$$y = x_{III} - x_I, \qquad x_{I,III} = (1-b)x_I + bx_{III} \tag{6.1a}$$

$$X = (1-a)x_{I,III} + ax_{II}, \qquad x = x_{II} - x_I \tag{6.1b}$$

where a and b are constants. In the $x_{III} \to x_I$ limit, (6.1) reduces to the form of (3.2). Similar to (I5.1), the plane wave ansatz in the laboratory frame is made

$$\chi_{\dot{b}\dot{h}}{}^f(x_I, x_{III}, x_{II}) = e^{-iK_\mu X^\mu} \chi_{\dot{b}\dot{h}}{}^f(x, y) \tag{6.2}$$

where K_μ now refers to baryons. Solutions in the relative times x^0 and y^0 of the form

$$\chi_{\dot{b}\dot{h}}{}^f(x, y) = \exp[i(\omega_0 x^0 + \omega_{0y} y^0)] \chi_{\dot{b}\dot{h}}{}^f(\mathbf{x}, \mathbf{y}) \tag{6.3}$$

are sought, analogous to (I6.5). Corresponding expressions for ψ analogous to (6.2)–(6.3) also hold. To remove the relative time dependences from the time derivatives, we make the choice

$$a = d + \omega_0/E_0, \qquad b = c + \omega_{0y}/E_0(1-d) \tag{6.4}$$

corresponding to the choice of a_m in Section 3. Here, c and d are constants.

Further treatment is confined to the rest frame $\mathbf{K} = 0$, as in Section 5 of II. In the absence of the external electromagnetic field A, (5.1) and (5.4) are considered, parallel to Section 3, as zeroth-order equations which have been treated in this frame in the $x_{III} \to x_I$ and $z_{III} \to z_I$ limits in II. The radial equations for the ground-state, spin-1/2 doublets and spin-3/2 quartets as well as their solutions at 0 and ∞ relative distances have been obtained. Since no data on spin-3/2 baryon magnetic moment are available, only those components of (6.3) associated with the ground-state, spin-1/2 doublets are of interest, i.e.,

$$\chi_{\dot{1}\dot{1}}{}^2 = -2\chi_{\dot{1}\dot{2}}{}^1 = -2\chi_{\dot{2}\dot{1}}{}^1 \tag{6.5a}$$

$$\chi_{\dot{2}\dot{2}}{}^1 = -2\chi_{\dot{1}\dot{2}}{}^2 = -2\chi_{\dot{2}\dot{1}}{}^2 \tag{6.5b}$$

according to (II2.6), putting the spin-3/2 quartet components to zero. Analogous expressions hold for ψ.

7. BARYON MAGNETIC MOMENT

Let

$$E = E_0 + E_{1B} \tag{7.1}$$

where E_0 is now the rest mass of the ground-state, spin-1/2 baryons and E_{1B} is due to the electromagnetic field A and hence is also of first order. The same simple choice of A in Section 4 is also made here.

Equation (5.4) is put in a form analogous to (4.2):

$$\begin{aligned}
&[i(1-d)(1-c)(E_0+E_{1B})\delta_{\dot{c}a} + (1-a)(1-b)\partial_{\mathbf{X}\dot{c}a} \\
&\quad + iq_{\mathrm{I}}A_{\dot{c}a}(\mathbf{X}) - (1-b)\partial_{\dot{c}a} - \partial_{y\dot{c}a}] \\
&\quad \times [i(1-d)(1-c)(E_0+E_{1B})\delta^{a\dot{b}} + (1-a)(1-b)\partial_{\mathbf{X}}^{a\dot{b}} \\
&\quad + iq_{\mathrm{I}}A^{a\dot{b}}(\mathbf{X}) - (1-b)\partial^{a\dot{b}} - \partial_y^{a\dot{b}}] \\
&\quad \times [i(1-d)c(E_0+E_{1B})\delta_{\dot{d}g} + (1-a)b\partial_{\mathbf{X}\dot{d}g} \\
&\quad + iq_{\mathrm{III}}A_{\dot{d}g}(\mathbf{X}) - b\partial_{\dot{d}g} + \partial_{y\dot{d}g}] \\
&\quad \times [i(1-d)c(E_0+E_{1B})\delta^{g\dot{h}} + (1-a)b\partial_{\mathbf{X}}^{g\dot{h}} \\
&\quad + iq_{\mathrm{III}}A^{g\dot{h}}(\mathbf{X}) - b\partial^{g\dot{h}} + \partial_y^{g\dot{h}}] \\
&\quad \times [id(E_0+E_{1B})\delta^{k\dot{e}} + ad_{\mathbf{X}}^{k\dot{e}} + iq_{\mathrm{II}}A^{k\dot{e}}(\mathbf{X}) + \partial^{k\dot{e}}] \\
&\quad \times [id(E_0+E_{1B})\delta_{\dot{e}f} + a\partial_{\mathbf{X}\dot{e}f} + iq_{\mathrm{III}}A_{\dot{e}f}(\mathbf{X}) + \partial_{\dot{e}f}]\chi_{\dot{b}\dot{h}}{}^{f}(\mathbf{x},\mathbf{y}) \\
&= -(M_b^3 + \phi_S(\mathbf{x},\mathbf{y}))^2\chi_{\dot{c}\dot{d}}{}^{k}(\mathbf{x},\mathbf{y}) + R_B e^{i(E_0+E_{1B})x^0}
\end{aligned} \tag{7.2}$$

where ∂_y refers to differentiation with respect to $\mathbf{y}$ and a reduction similar to that leading from (II2.10) to (II5.3) has been made for ϕ_S.

By analogy to $\omega_0 = 0$ in Section 4, the relative energies ω_0 and ω_{0y} are assumed to vanish in the rest frame. This assumption can perhaps be removed in a quantized treatment, analogous to that for mesons (Hoh, 1994*b*). These relative energies in the $\partial_{\mathbf{X}}$ terms in (7.2) now drop out. Collecting terms to the first order in E_{1B} in (7.2), we can write the counterpart of (4.3a) as

$$\begin{aligned}
&-6E_{1B}E_0(1-d)^4d^2(1-c)^2c^2\Bigg\{E_0^4 + \frac{2}{3}\left[2\left(\frac{1}{1-d}\right)^2 + \frac{1}{d^2}\right]E_0^2\Delta \\
&+\frac{1}{3}\left[2\left(\frac{1}{(1-d)d}\right)^2 + \left(\frac{1}{1-d}\right)^4\right]\Delta\Delta\Bigg\}\begin{pmatrix}\chi_{\dot{1}\dot{1}}{}^{2}(\mathbf{x}) \\ \chi_{\dot{2}\dot{2}}{}^{1}(\mathbf{x})\end{pmatrix}
\end{aligned} \tag{7.3a}$$

The corresponding terms to first order in A in (7.2) that split (6.5a) from

(6.5b) are found to be

$$(cq_{\rm I}+(1-c)q_{\rm III})H_3(\mathbf{X})(1-d)^3d^2(1-c)c\left\{E_0^4+\left[\left(\frac{1}{1-d}\right)^2+\frac{1}{d^2}\right]E_0^2\Delta + \left(\frac{1}{(1-d)d}\right)^2\Delta\Delta\right\}\begin{pmatrix}\chi_{\rm 1\dot{1}}{}^2(\mathbf{x})\\ \chi_{\dot{2}\dot{2}}{}^1(\mathbf{x})\end{pmatrix} - q_{\rm II}H_3(\mathbf{X})(1-d)^4d(1-c)^2c^2$$
$$\times\left[E_0^4+\frac{2}{(1-d)^2}E_0^2\Delta+\left(\frac{1}{1-d}\right)^4\Delta\Delta\right]\begin{pmatrix}\chi_{\rm 1\dot{1}}{}^2(\mathbf{x})\\ -\chi_{\dot{2}\dot{2}}{}^1(\mathbf{x})\end{pmatrix} \tag{7.3b}$$

Here, E_0 is the eigenvalue of $i\partial/\partial X^0$ with X defined in (6.1b), which differs from the definition of X in Section 5 of II, which is of the form (3.2). However, these two definitions of X differ only by a constant and hence lead to the same E_0. Further, χ is the zeroth-order ground-state baryon wave function in relative space, i.e., (6.5) with $y\to 0$ in (6.1a). By (II2.6) and (II6.3) with $l=0$, it becomes

$$\chi_{\rm 1\dot{1}}{}^2(\mathbf{x}) = -\frac{4}{3}\chi_{0\dot{1}} = -\frac{4}{3}\left(g_0(r)Y_{00}+\frac{i}{\sqrt{3}}f_0(r)Y_{10}\right) \tag{7.4a}$$

$$\chi_{\dot{2}\dot{2}}{}^1(\mathbf{x}) = \frac{4}{3}\chi_{0\dot{2}} = \frac{4}{3}\left(-g_0(r)Y_{00}+\frac{i}{\sqrt{3}}f_0(r)Y_{10}\right) \tag{7.4b}$$

where Y denotes normalized spherical harmonics, $r=|\mathbf{x}|$, and f_0 and g_0 are radial wave functions of the ground-state, spin-1/2 baryons determined by (II6.8)–(II6.9).

Multiplying the upper and lower rows of (7.3) by $(\chi_{\rm 1\dot{1}}{}^2(\mathbf{x}))^*$ and $(x_{\dot{2}\dot{2}}{}^1(\mathbf{x}))^*$, respectively, and integrating over $\mathbf{x}$ yields the ground-state, spin-1/2 baryon magnetic moment:

$$\mu_B = \frac{1}{2E_0}\frac{1}{3}\left[\frac{1}{1-d}\left(\frac{q_{\rm I}}{1-c}+\frac{q_{\rm III}}{c}\right)k_{\rm I}-\frac{1}{d}q_{\rm II}k_{\rm II}\right]\Big/k_E \tag{7.5}$$

$$k_E = 1+\frac{2}{3}\left(\frac{2}{(1-d)^2}+\frac{1}{d^2}\right)\kappa_1+\frac{1}{3}\left[2\left(\frac{1}{(1-d)d}\right)^2+\left(\frac{1}{1-d}\right)^4\right]\kappa_2 \tag{7.6a}$$

$$k_{\rm I} = 1+\left[\left(\frac{1}{1-d}\right)^2+\frac{1}{d^2}\right]\kappa_1+\left(\frac{1}{(1-d)d}\right)^2\kappa_2 \tag{7.6b}$$

$$k_{\rm II} = 1+\frac{2}{(1-d)^2}\kappa_1+\frac{1}{(1-d)^4}\kappa_2 \tag{7.6c}$$

$$\kappa_1 = \int d^2\mathbf{x}\,(\chi_{\rm 1\dot{1}}{}^2(\mathbf{x}))^*\Delta\chi_{\rm 1\dot{1}}{}^2(\mathbf{x})/E_0^2\int d^3\mathbf{x}|\chi_{\rm 1\dot{1}}{}^2(\mathbf{x})|^2 \tag{7.7a}$$

$$\kappa_2 = \int d^3\mathbf{x}\,(\chi_{\rm 1\dot{1}}{}^2(\mathbf{x}))^*\Delta\Delta\chi_{\rm 1\dot{1}}{}^2(\mathbf{x})/E_0^4\int d^3\mathbf{x}\,|\chi_{\rm 1\dot{1}}{}^2(\mathbf{x})|^2 \tag{7.7b}$$

The coordinate transformation constants c and d associated with the quark coordinates, which are not observable, are in principle arbitrary. Here, however, they may be determined from data or possibly from an energy minimization procedure similar to what follows.

In (7.6), the departure of k from unity signifies the effects of quark motion. For the hypothetical case of quark at rest, $\Delta \to 0$ and (7.5) degenerates into

$$\mu_{B0} = \frac{1}{2E_0}\frac{1}{3}\left[\frac{1}{1-d}\left(\frac{q_{\rm I}}{1-c} + \frac{q_{\rm III}}{c}\right) - \frac{q_{\rm II}}{d}\right] \tag{7.8}$$

Let (7.3a) in the absence of the Δ terms be maximized so that E_{1B} is minimized. This leads to

$$d = 1/3, \qquad c = 1/2 \tag{7.9}$$

which also puts $q_{\rm I}$, $q_{\rm II}$, and $q_{\rm III}$ on an equal footing. This "static quark" approximation of the baryon magnetic moment is then

$$\mu_{B0m} = \frac{1}{2E_0}(q_{\rm I} + q_{\rm III} - q_{\rm II}) \tag{7.10}$$

8. FURTHER EVALUATION OF BARYON MAGNETIC MOMENT

To simplify notation, let $z_{\rm I} = z$, $z_{\rm III} = v$, $z_{\rm II} = u$. The normalized internal functions for ground-state, spin-1/2 baryons are (Lichtenberg, 1978; Carruthers, 1967)

$$\begin{aligned}
\xi(p) &= \xi^{112} = \frac{1}{\sqrt{6}}(2z^1v^1u^2 - z^1v^2u^1 - z^2v^1u^1) \\
\xi(n) &= \xi^{221} \\
\xi(\Sigma^+) &= \xi^{113}, \qquad \xi(\Sigma^-) = \xi^{223} \\
\xi(\Sigma^0) &= \xi^{123} = \frac{1}{\sqrt{12}}(2z^1v^2u^3 + 2z^2v^1u^3 - z^2v^3u^1 \\
&\quad - z^3v^2u^1 - z^1v^3u^2 - z^3v^1u^2) \\
\xi(\Lambda) &= \xi_\Lambda^{123} = \frac{1}{2}(z^2v^3u^1 + z^3v^2u^1 - z^1v^3u^2 - z^3v^1u^2) \\
\xi(\Xi^0) &= \xi^{331}, \qquad \xi(\Xi^-) = \xi^{332}
\end{aligned} \tag{8.1}$$

For the cases in (8.1), (5.2) can be evaluated, making use of (2.8) together with the $\rm I \to II$ and III cases and of (2.7), generalized to include $z_{\rm III}$. The results are given in Table I.

Table I. $q_{\rm I}$, $q_{\rm II}$, $q_{\rm III}$ for Octet Baryons and the "Static Quark" Approximation of Baryon Magnetic Moments (7.10) in Proton Magnetons

	p	n	Λ	Σ^+	Σ^0	$\Sigma^0 \to \Lambda$	Σ^-	Ξ^0	Ξ^-
$q_{\rm I}/e = q_{\rm III}/e$	1/2	−1/6	−1/12	1/2	1/12	$-1/4\sqrt{3}$	−1/3	−1/6	−1/3
$q_{\rm II}/e$	0	1/3	1/6	0	−1/6	$1/2\sqrt{3}$	−1/3	1/3	−1/3
μ_{B0m} in (7.10)	1	−2/3	−0.27	0.789	0.262	0.454	0.261	−0.476	−0.237

With (7.4), (7.7) can be put in the form

$$\kappa_1 = \frac{8}{N_{cd}} \int_0^\infty dr \cdot r^2 \left(g_0(r)\Delta_0 g_0(r) + \frac{1}{3} f_0(r)\Delta_1 f_0(r) \right) \tag{8.2a}$$

$$\kappa_2 = \frac{8}{N_{cd}} \int_0^\infty dr \cdot r^2 \left(g_0 \Delta_0 \Delta_0 g_0 + \frac{1}{3} \Delta_1 \Delta_1 f_0(r) \right) \Big/ E_0^2 \tag{8.2b}$$

$$\Delta_l = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{l(l+1)}{r^2} \tag{8.2c}$$

where N_{cd} is the conserved quantity given by (II6.9c):

$$N_{cd} = 8E_0^2 \int_0^\infty dr \cdot r^2 \left(g_0^2(r) + \frac{1}{3} f_0^2(r) \right) \tag{8.2d}$$

The octet baryon magnetic moment is now given by (7.5), (7.6), (8.2), together with Table I. It can presently not be evaluated, apart from the values of c and d discussed below (7.7), because the radial wave functions f_0 and g_0 in (8.2) have not been worked out presently. These functions depend upon the quark masses and hence vary among the octet members of (8.1).

If (7.9) is adopted, (7.5) and (7.6) can, noting $q_{\rm I} = q_{\rm III}$, be simplified to

$$\mu_B = \frac{1}{2E_0} \frac{2q_{\rm I}(1 + (45/4)\kappa_1 + (81/4)\kappa_2) - q_{\rm II}(1 + (9/2)\kappa_1 + (81/16)\kappa_2)}{1 + 9\kappa_1 + (81/2)\kappa_2} \tag{8.3}$$

If quark motion is neglected, $\kappa_1 = \kappa_2 = 0$, and (8.3) reduces to (7.10), which in units of the proton magneton is given by the last line in Table I. These values are roughly 1/3 of the measured ones and hence also of the predicted values of the nonrelativistic model (Lichtenberg, 1978). This $\sim 1/3$ factor stems from the fact that the quark masses in the Dirac magnetic moment used in the literature are replaced by the baryon mass in (7.10).

In summary, the present approach and results differ from those of the literature (Lichtenberg, 1978) mainly in two aspects. In the first place, the baryon magnetic moment arises from operation in the internal or z space (5.2) together with (8.1), contrary to operations in spin space. This makes it possible for the same spin space symmetry to be assigned to all ground-state baryons, contrary to the requirement that the Λ has a symmetry different from the other seven members of the octet (Lichtenberg, 1978). Second, the present hadron magnetic moment formulas (4.6), (7.5), (7.6), and (8.2) are derived from the basic covariant equations in I and II through a series of specializations and without any approximation of significance, exc ͺpt for the assumption of zero relative energies above (7.3a). As was mentioned there, such an assumption was supported in the meson case and can perhaps be removed in a quantized treatment. The formulas here thus include relativistic effects of quark motion, unlike the phenomenological and nonrelativistic treatment in the literature.

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